TEST INFORMATION

DATE : 15.04.2015   **PART TEST-01 (PT-01)**

Syllabus : Function & Inverse Trigonometric Function, Limits, Continuity & Derivability, Quadratic Equation

REVISION DPP OF FUNCTION AND INVERSE TRIGONOMETRIC FUNCTION

Total Marks : 171   **Max. Time : 151 min.**

Single choice Objective (no negative marking) Q. 1 to 10	**(3 marks 3 min.)**	**[30, 30]**
Multiple choice objective (no negative marking) Q. 11 to 32	**(5 marks, 4 min.)**	**[110, 88]**
Comprehension (no negative marking) Q.33 to 37	**(3 marks 3 min.)**	**[15, 15]**
Match the Following (no negative marking) Q.38	**(8 marks, 8 min.)**	**[8, 8]**
Subjective Questions (no negative marking) Q. 39,40	**(4 marks 5 min.)**	**[8, 10]**

1. If $e^x + e^{f(x)} = e$, then the range of f(x) is
(A) $(-\infty, 1]$ (B) $(-\infty, 1)$ (C) $(1, \infty)$ (D) $[1, \infty)$

2. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\left(\cos\frac{7\pi}{5} - \sin\frac{2\pi}{5}\right)\right)$ is equal to
(A) $\frac{23\pi}{20}$ (B) $\frac{13\pi}{20}$ (C) $\frac{3\pi}{20}$ (D) $\frac{17\pi}{20}$

3. Number of solutions of equation $3 + [x] = \log_2(9 - 2^{\{x\}}) + x$, $x \in [-1, 4]$ where [x] and {x} denote integral and fractional part of x respectively, is
(A) 6 (B) 12 (C) 2 (D) 1

4. If f(x) = x + sinx then all points of intersection of y = f(x) and $y = f^{-1}(x)$ lie on the line
(A) y = x (B) y = –x (C) y = 2x (D) y = –2x

5. Range of $f(\theta) = \tan\left(\operatorname{cosec}^{-1}\left(\frac{1}{\sqrt{2}\sin\theta}\right)\right)$ is
(A) $(-\infty, \infty) - \{n\pi\}$ (B) $R - \{0\}$
(C) $[0, \infty)$ (D) $(-\infty, -\sqrt{2}\,] \cup \{0\} \cup [\sqrt{2}, \infty)$

6. P(x) is a polynomial of degree 98 such that $P(K) = \frac{1}{K}$ for K = 1, 2, 3, ……. 99. The value of P(100) is
(A) $\lfloor 100 + 1$ (B) $\frac{1}{100}$ (C) $\frac{1}{50}$ (D) $\frac{1}{\lfloor 100}$

7. For each positive integer n, let $f(n + 1) = n(-1)^{n+1} - 2f(n)$ and f(1) = f(2010). Then $\sum_{K=1}^{2009} f(K)$ is equal to
(A) 335 (B) 336 (C) 331 (D) 333

8. If f(x) = x + tanx and f(x) is inverse of g(x), then g'(x) is equal to
(A) $\frac{1}{1+(g(x)-x)^2}$ (B) $\frac{1}{1+(g(x)+x)^2}$ (C) $\frac{1}{2-(g(x)-x)^2}$ (D) $\frac{1}{2+(g(x)-x)^2}$

9. Number of solution of the equation $\tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right) = \sqrt{1-x^2}$ is equal to
(A) 0 (B) 1 (C) 2 (D) 3

10. If x and y are of same sign, then the value of $\frac{x^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\frac{x}{y}\right)+\frac{y^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\frac{y}{x}\right)$ is equal to

(A) $(x - y)(x^2 + y^2)$ (B) $(x + y)(x^2 - y^2)$ (C) $(x + y)(x^2 + y^2)$ (D) $(x - y)(x^2 - y^2)$

11. For $f(x) = \tan^{-1}\left(\frac{\left(\sqrt{12}-2\right)\ x^2}{x^4+2x^2+3}\right)$

(A) $f_{max} = \frac{\pi}{12}$ (B) $f_{min} = 0$ (C) f_{min} does not exist (D) $f_{max} = \frac{\pi}{2}$

12. If $f(x) = \begin{cases} x+1, & x \le 0 \\ 2-x, & x > 0 \end{cases}$ and $g(x) = \begin{cases} x^2+1, & x \ge 1 \\ 2x-3, & x < 1 \end{cases}$ then

(A) Range of gof (x) is $(-\infty, -1) \cup [2, 5]$ (B) Range of gof (x) is $(-\infty, -1) \cup [2, 5)$
(C) gof (x) is one-one for $x \in [0, 1]$ (D) gof (x) is many one for $x \in [0, 1]$

13. If f(x) is identity function, g(x) is absolute value function and h(x) is reciprocal function then
(A) fogoh(x) = hogof(x) (B) hog(x) = hogof(x)
(C) gofofofohogof(x) = gohog(x) (D) hohohoh(x) = f(x)

14. The function $y = \frac{x}{1+|x|} : R \to R$ is

(A) one-one (B) onto (C) odd (D) into

15. If α, β, γ are roots of equation $\tan^{-1}(|x^2 + 2x| + |x + 3| - ||x^2 + 2x| - |x + 3||) + \cot^{-1}\left(-\frac{1}{2}\right) = \pi$ in ascending order $(\alpha < \beta < \gamma)$ then
(A) $\sin^{-1}\gamma$ is defined (B) $\sec^{-1}\alpha$ is defined
(C) $\gamma - \beta = \sqrt{2}$ (D) $|\beta| > |\gamma|$

16. If f(x) and g(x) are two polynomials such that the polynomial $h(x) = xf(x^3) + x^2g(x^6)$ is divisible by $x^2 + x + 1$, then
(A) f(1) = g(1) (B) f(1) = –g(1) (C) h(1) = 0 (D) all of these

17. $1 + [\sin^{-1}x] > [\cos^{-1}x]$ where [.] denotes GIF, if $x \in$
(A) (cos1, sin1) (B) [sin1, 1] (C) (cos1, 1] (D) [cos1, 1]

18. If the solution of equation $\sin(\tan^{-1}x) = \sqrt{4-\left[\sin\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}x\right)\right]^2}$ is a, then

(A) $\sin^{-1}a + \cos^{-1}a = \frac{\pi}{2}$ (B) $2\sin^{-1}a + \cos^{-1}a = \frac{\pi}{2}$ (C) $\sin^{-1}a + 3\cos^{-1}a = \frac{3\pi}{2}$ (D) $\tan^{-1}a + \cos^{-1}a = \frac{\pi}{2}$

19. If $f(x) = \frac{2^{\{x\}}-1}{2^{\{x\}}+1}$ then (where {x} represent fractional part of x)

(A) $D_f \in R$ (B) $R_f \in [0, \frac{1}{3})$ (C) period of f(x) is 1 (D) f(x) is even function

20. Which of the following is true for $f(x) = (\cos x)^{\cos x}$, $x \in \left[-\cos^{-1}\frac{1}{e}, \cos^{-1}\frac{1}{e}\right]$

(A) $R_f \in \left[\left(\frac{1}{e}\right)^{1/e}, 1\right]$ (B) f(x) is increasing (C) f(x) is many-one (D) f(x) is maximum at x = 0

21. If $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is a bijective function from set A to set B then which of the following may be true

(A) $A = (-\infty, -1)$, $B = \left(0, \frac{\pi}{2}\right)$ (B) $A = (-1, 1)$, $B = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $A = [1, \infty)$, $B = \left(-\frac{\pi}{2}, 0\right]$ (D) All of these

22. If the functions f(x) and g(x) are defined from R^+ to R such that

$f(x) = \begin{cases} 1-\sqrt{x} & ; \text{ x is rational} \\ x^2 & ; \text{ x is irrational} \end{cases}$ and $g(x) = \begin{cases} x & ; \text{ x is rational} \\ 1-x & ; \text{ x is irrational} \end{cases}$, then the composite function fog(x) is

(A) one – one (B) many-one (C) into (D) onto

23. Let $f(x) = ([a]^2 - 5[a] + 4)x^3 + (6\{a\}^2 - 5\{a\} + 1)x - \tan x.\text{sgn}(x)$ is an even function for all $x \in R$, where [.] and {.} are greatest integer and fractional part functions respectively, then which of the following is defined

(A) $\sin^{-1}a$ (B) $\tan^{-1}a$ (C) $\sec^{-1}a$ (D) $\sqrt[3]{a-2}$

24. Let $f(x) = \cot^{-1}(x^2 + 4x + \alpha^2 - 3\alpha)$ be a function defined on $R \to \left(0, \frac{\pi}{2}\right]$, is an onto function then

(A) $\alpha \in [-1, 4]$ (B) $f'(0) = -4/17$ (C) f(x) is one-one (D) f(x) is many-one

25. The number of solutions of equation $2\cos^{-1}x = a + a^2(\cos^{-1}x)^{-1}$ are

(A) at least 1 if $a \in [-2\pi, \pi] - \{0\}$ (B) 1 if $a \in (0, \pi]$
(C) 1 if $a \in [-2\pi, 0)$ (D) 2 if a > 0

26. The function $f : \left[-\frac{1}{2}, \frac{1}{2}\right] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is

(A) a surjective function (B) an injective function
(C) a surjective but not injective (D) neither injective nor surjective

27. If $f(x) = \left[\frac{1}{\ell n(x^2 + e)}\right] + \frac{1}{1+x^2}$ where [.] is greatest integer function then

(A) $f(x) \in \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \cup \{2\}$ for $x \in R-\{1\}$ (B) $R_f = (0, 1) \cup \{2\}$
(C) f is many-one (D) f(x) is bounded

28. If $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$, then $\underbrace{\left(h\left(h\left(h......\left(h(x)\right)\right)\right)\right)}_{\text{h repeated n times}}$ is

(A) identity function (B) one-one (C) odd (D) periodic

29. The function $f : R \to (-1, 1)$ is defined by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

(A) f(x) is a bijective function (B) f(x) is non–bijective function
(C) $f^{-1}(x) = \frac{1}{2}\ell n\left(\frac{1+x}{1-x}\right)$ (D) f(x) is many one onto function

30. Which of the following is true?

(A) $2\tan^{-1}x = \pi - \sin^{-1}\frac{2x}{1+x^2}$ if x > 1 (B) $\tan^{-1}\frac{1}{x} = -\pi + \cot^{-1}x$ if x < 0
(C) $\sec^{-1}x = \sin^{-1}\frac{\sqrt{x^2-1}}{x}$ if |x| > 1 (D) $\sin(\tan^{-1}(\text{cosec}(\cos^{-1}x))) = \frac{1}{\sqrt{2-x^2}}$ if –1 < x < 0

31. Let $f:[a, \infty) \to [a, \infty)$ be a function defined by $f(x) = x^2 - 2ax + a(a + 1)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 2014, then the other solution may be

(A) 2013 (B) 2015 (C) 2016 (D) 2012

32. Let $f(x) = \frac{3}{4}x + 1$ and $f^{n+1}(x) = f(f^n(x))\ \forall n \geq 1, n \in N$. If $\lim_{n\to\infty} f^n(x) = \lambda$, then

(A) λ is independent of x.
(B) λ is a linear polynomial in x.
(C) line $y = \lambda$ has slope 0.
(D) line $4y = \lambda$ touches a circle of unit radius with centre at origin.

Comprehension # 1 (Q. no. 33 to 35)

Let f : $[2, \infty) \to [1, \infty)$ defined by f(x) = $2^{x^4-4x^2}$ and g : $\left[\frac{\pi}{2},\pi\right] \to A$, defined by g(x) = $\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

33. $f^{-1}(x)$ is equal to

(A) $-\sqrt{2+\sqrt{4+\log_2 x}}$ (B) $\sqrt{2+\sqrt{4+\log_2 x}}$ (C) $\sqrt{2-\sqrt{4+\log_2 x}}$ (D) $-\sqrt{2-\sqrt{4+\log_2 x}}$

34. The set A is equal to

(A) [–5, –2] (B) [2, 5] (C) [–5, 2] (D) [–3, –2]

35. Domain of $fog^{-1}(x)$ is

(A) [–5, sin1] (B) $\left[-5,\frac{\sin 1}{2-\sin 1}\right]$ (C) $\left[-5,\frac{4+\sin 2}{\sin 2-2}\right]$ (D) $\left[\frac{4+\sin 2}{\sin 2-2},-2\right]$

Comprehension # 2 (Q. no. 36 to 37)

Let $f(x) = x^2 + xg'(1) + g''(2)$ and $g(x) = f(1)\, x^2 + xf'(x) + f''(x)$.

36. The domain of function $\sqrt{\frac{f(x)}{g(x)}}$ is

(A) $(-\infty, 1] \cup (2, 3]$ (B) $(-2, 0] \cup (1, \infty)$ (C) $(-\infty, 0] \cup \left(\frac{2}{3},\ 3\right]$ (D) None of these

37. Area bounded between the curves y = f(x) and y = g(x) is

(A) $\frac{4\sqrt{2}}{3}$ (B) $\frac{8\sqrt{2}}{3}$ (C) $\frac{2\sqrt{2}}{3}$ (D) $\frac{16}{3}$

38. **Match the columns :**

Let f(x) = log(secx), g(x) = f′(x) and 'n' is an integer.

Column – I		Column-II
(A) Domain of f(x) is	(p)	$\bigcup_{n\in I}\left(2n\pi-\frac{\pi}{2},2n\pi+\frac{\pi}{2}\right)$
(B) Domain of g(x) is	(q)	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$
(C) If fundamental period of g(x) is k then k is element of set	(r)	$\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$
(D) gog^{-1} is an identity for x ∈	(s)	$\left(\frac{3\pi}{2},\frac{5\pi}{2}\right)$

39. Let $f(x) = -4\sqrt{e^{1-x}} + 1 + x + \frac{x^2}{2}+\frac{x^3}{3}$. If g(x) is inverse of f(x), then find the value of reciprocal of $g'\left(-\frac{7}{6}\right)$.

40. Let f : $R^+ \to R^+$ be a function which satisfies the relation $f(x).f(y) = f(xy) + 2\left(\frac{1}{x}+\frac{1}{y}+1\right)$ then find the value of $f\left(\frac{1}{2}\right)$.